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example of uncorrelated but dependent
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Hi John,

In case you're interested, I started with two normally-distributed random variables $X,Y \sim N(0,1)$ then applied the following transformation to generate new random variables U,V:

$U = \{ |X+Y|, -|X+Y| \}$
$V = \{ X+Y, -(X+Y) \}$

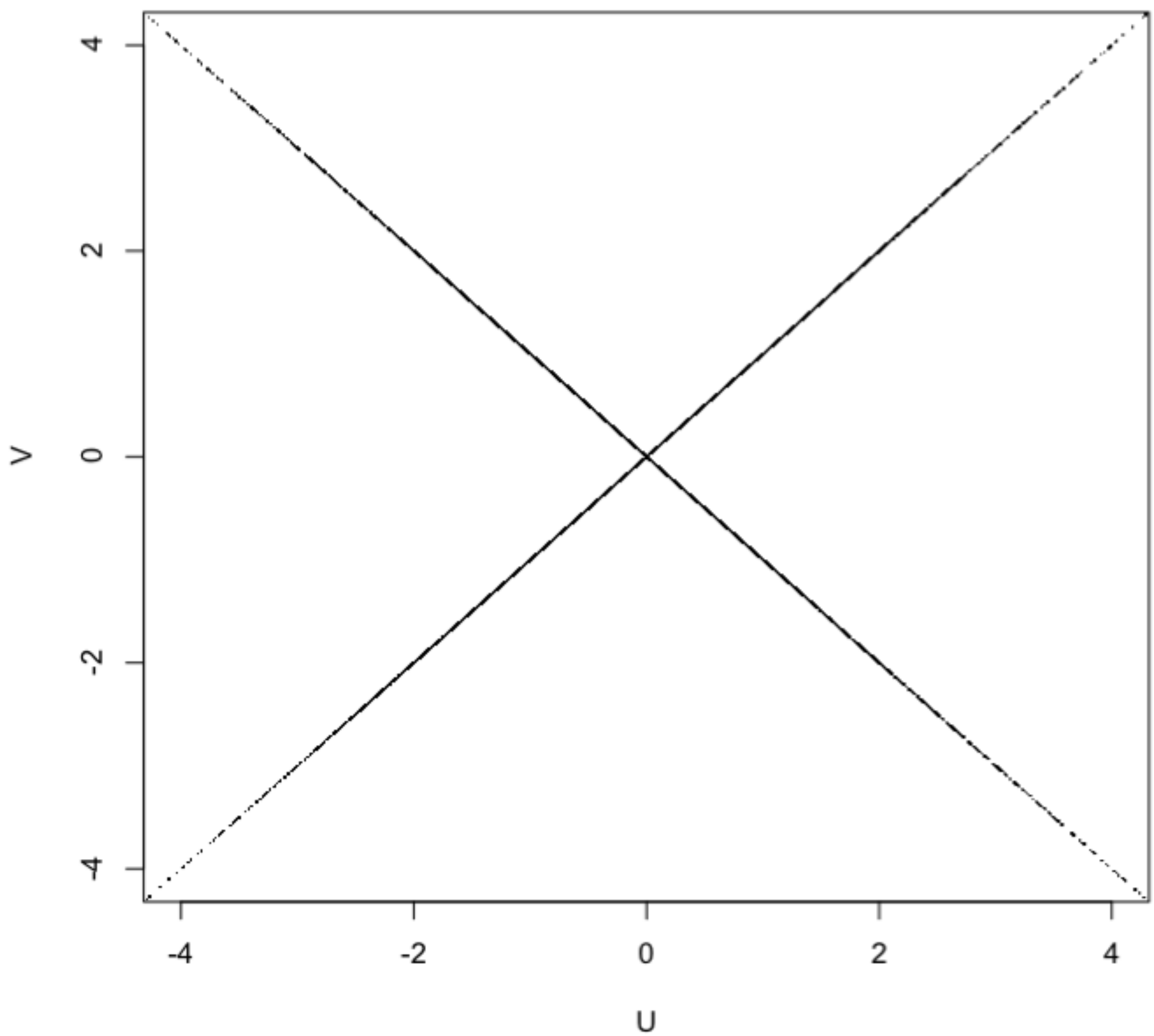
The notation here means that the vector formed by $|X+Y|$ is joined with $-|X+Y|$ to make a longer vector (= U) etc..

This transformation leads to marginalized *normal* distributions in each of U and V (also if projected onto each axis) and $\mathrm{cov}(U,V) = 0$, i.e., they are uncorrelated. However **U and V are dependent** (see figure below).

Therefore, this is any example where two variables (U,V) are **not** joint-normal, have covariance = 0 but are indeed dependent. It reinforces the fact that the covariance measure completely determines (in)dependency between variables if and only if they are joint-normally distributed.

As a side note, joint-normally distributed variables can only be **linearly-dependent**. That's because ellipses and ellipsoids are the 'norm' for multinormal distributions. This is what the covariance can ever measure for joint-normally distributed data. E.g., Pearson's product moment coefficient is sufficient for computing the correlation between normally distributed variables because it measures the degree of linear-dependence only. For a more general measure of dependency between variables (e.g., related to high polynomial order), Spearman's rank correlation coefficient is the best choice.

Regards, Frank



V
4
2
0
-2
-4
-4
-2
0
2
4
U