From: Frank Masci <fmasci@ipac.caltech.edu>
Subject: Re: impact of correlations for N>=3
Date: November 2, 2011 1:26:58 PM PDT
To: John Fowler <jwf@ipac.caltech.edu>

Hi John,

I found some time to look at this and I do see why there's a paradox. A transformation that involves just rotating a collection of random vectors will always result in a joint (bivariate) normal distribution for u and v. In fact, this is true for any linear transformation on x,y. In your case, the covariance = 0 because the variances of x,y are equal (in fact, these are also equal to the variances of u and v for a pure rotation). In this case, the bivariate density function is separable. You're just rotating a point on the same circle. Given that u,v satisfy joint normality with covariance = 0, they're classified as independent.

Furthermore are u and v really dependent on each other (i.e., to create a pattern in the u-v plane)? They're surely related by the constraint u^2 + v^2 = 1, but does this qualify as a dependence?

So, my claim is that if you can construct a plausible multivariate normal distribution with the new variables, then the notion of dependence vs. independence is exclusively determined by their covariances. If you cannot construct one, e.g., looking at the covariance structure should suffice, the variables could turn out to dependent but uncorrelated according to Pearson's rho.

Also, the following site may be of interest:
http://en.wikipedia.org/wiki/Normally_distributed_and_uncorrelated_does_not_imply_independent

Regards, Frank

On Oct 30, 2011, at 11:09 PM, John Fowler wrote:

> One other puzzlement: on October 8 I said (not entirely correctly):
> <dbdchabj.png>
>
> My point was that u and v are not independent because they are linear combinations

of the same two other variables, X and Y. And yet apparently they can be independent despite that! If X and Y are Gaussian, then so are u and v, since the latter are linear combinations of the former. But two Gaussian random variables that are uncorrelated are also independent, because the joint density function separates into the product of the two marginal density functions, and that is the definition of independence. I still haven't come to terms with u and v each being functions of the same two variables and yet still being independent nonetheless. Do you have a viewpoint of that paradox that makes sense to your intuition?

Regards, John